Exam Seat No:________________ Enrollment No:___________________

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code 4TE02EMT1

Summer Examination-2014 **Date: 26/05/2014**

Subject Name: Engineering Mathematics-II

Branch/Semester:- B.Tech/II **Time:02:00 To 5:00**

Examination: Regular

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places
(5) Assume suitable & Perfect data if needed

SECTION - I

Q-1 (a) Find the determinant of the matrix A, Where $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 5 & 7\end{bmatrix}$ [01]

(b) Evaluate: $\int_{-\pi/2}^{\pi/2} \sin^6 x \ dx$ [02]

(c) Define : Symmetric matrix and Hermitian matrix with suitable examples. [02]

(d) Find the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^3+\left(\frac{d^2y}{dx^2}\right)^4+y=0$ [02]

Q-2 (a) (i) Find the differential equation of all circles of radius r whose centers lie on the x-axis. [02]

(ii) Solve : $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x^2}$ [03]

(b) Evaluate : $\int_2^\infty \frac{x+3}{(x-1)(x^2+1)}dx$ [05]

(c) Solve the following system of equations by Cramer's Rule. [04]

$-\frac{1}{x}+\frac{3}{y}+\frac{4}{z}=30\ ,\ \frac{3}{x}+\frac{2}{y}-\frac{1}{z}=9\ ,\ \frac{2}{x}-\frac{1}{y}+\frac{2}{z}=10$

OR

Q-2 (a) (i) Solve : $x\,dx+y\,dy+y^2\,dx=0$ [02]

(ii) Evaluate : $\int_0^\infty \frac{x^2}{(1+x^2)^{\frac{7}{2}}}\,dx$ [03]

(b) Find the volume of solid of revolution, obtained by rotating the area bounded below the line $2x+3y = 6$ in the first quadrant, about the x-axis. [05]

(c) Find the matrix A, if $A^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ [04]

Q-3 (a) (i) Find the rank of the following matrix by row-echelon form. [03]

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

(ii) Discuss the consistency of the system and if consistent then solve the equation by Gauss-Elimination method.

$4x - 2y + 6z = 8$, $x + y - 3z = -1$, $15x - 3y + 9z = 21$ [04]

(b) Find the Eigenvalues & eigenvectors of the given matrix A. And also find algebraic multiplicity and geometric multiplicity of it. Where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ [07]

OR

Q-3 (a) (i) Obtain Row echelon & Reduced row echelon form of the following matrix: [03]

$$A = \begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

(ii) Solve the system of equations : [04]

$$2x_1 - x_2 + 3x_3 = 0$$
$$3x_1 + 2x_2 + x_3 = 0$$
$$x_1 - 4x_2 + 5x_3 = 0$$

(b) Find the characteristic roots of the matrix $A$and verify Cayley-Hamilton theorem for this matrix. Hence find A^{-1} and also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. Where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ [07]

SECTION - II

Q-4 (a) State the Cayley-Hamilton Theorem. [01]

(b) Evaluate : $\int_0^1 \int_0^x e^x \, dx \, dy$ [02]

(c) Prove that $\nabla r^2 = 2\bar{r}$ [02]

(d) Find the eigenvalue of A and A^7, Where $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$ [02]

Q-5 (a) (i) Find the constants *a*, *b*, *c* if $\bar{F} = \left(axy + bz^3\right)i + \left(3x^2 - cz\right)j + \left(3xz^2 - y\right)k$ is irrotational. [02]

(ii) Find the directional derivative of $\phi = e^{2x} \cos yz$ at the origin in the direction of the tangent to the curve $x = a\sin t, y = a\cos t, z = at$ at $t = \frac{\pi}{4}$. [03]

(b) Evaluate : $\oint_C \left[\left(x^2 - \cosh x\right)dx + \left(y + \sin x\right)dy\right]$ by Green's theorem. Where C is the rectangle with vertices $(0,0), (\pi,0), (\pi,1)$ and $(0,1)$. [05]

(c) Evaluate $\iint xy\left(x+y\right)dx\,dy$ over the region bounded by the parabolas $y^2 = x, x^2 = y$ [04]

OR

Q-5 (a) (i) Find the constant *a* if $\bar{F} = \left(x + 3y^2\right)i + \left(2y + 2z^2\right)j + \left(x^2 + az\right)k$ is solenoidal. [02]

(ii) Evaluate : $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\,dy}{1 + x^2 + y^2}$ [03]

(b) Evaluate $\int_C \bar{F} \cdot d\bar{r}$, where $\bar{F} = \left(2x + y^2\right)i + \left(3y - 4x\right)j$ and C is the triangle in the *xy*-plane with vertices $(0,0), (2,0)$ and $(2,1)$. [05]

(c) Change the order of integration and evaluate $\int_0^\infty \int_0^x xe^{-\frac{x^2}{y}} dy\,dx$. [04]

Q-6 (a) (i) Express the following matrix as the sum of a symmetric matrix and a skew symmetric matrix: $A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ [03]

(ii) Investigate for what values of λ and μ have (a) no solution, (b) a unique [04] solution and (c) infinitely many solutions.

$$x + 2y + z = 8$$
$$2x + 2y + 2z = 13$$
$$3x + 4y + \lambda z = \mu$$

(b) Find a matrix that diagonalizable and determine $P^{-1}AP$, where $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ [07]

OR

Q-6 (a) (i) Prove that matrix A is unitary matrix and hence find A^{-1}, [03]

where $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$

(ii) Find the inverse of the following matrix by elementary transformation: [04]

$$A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 2 & 1 \\ 3 & -2 & 1 & 6 \end{bmatrix}$$

(b) Find a matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. Hence find A^{13}. [07]

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